Online Supplementary Material

Properties of the Jovian atmosphere and vortices

To compute the values of the GRS and the Oval BA shown in table 1, we used $g = 25$ m.s^{-2}, a vertical pressure scale-height of 23 km, and $H = 46 \pm 14$ km (as described in section 5). For the GRS (de Pater *et al.* 2010; Shetty & Marcus 2010) we used $\bar{N} = 0.0158 \pm 0.0005$ rad/s, $V_\theta = 119 \pm 14$ m/s, $L = 2300 \pm 70$ km, $R_v = 6260 \pm 50$ km, and $f = 1.374 \times 10^{-4}$ rad/s. For the Oval BA we used $\bar{N} = 0.0182 \pm 0.0006$ rad/s, $V_\theta = 101 \pm 9$ m/s, $L = 3200 \pm 45$ km, $R_v = 3200 \pm 45$ km, and $f = 1.915 \times 10^{-4}$ rad/s (de Pater *et al.* 2010; Shetty & Marcus 2010).

Thermal imaging of the Jovian atmosphere gives the values of the temperature of the clouds at the elevations of the Great Red Spot (GRS) and Oval BA. In particular, the imaging gives the temperature $T_{c'}$ (where c' indicates the intersection of the vertical central rotation axis of a vortex and the the top of the vortex at $z = H$). Thermal imaging also gives the value of the background temperature $\bar{T}|_{z=H}$ of the atmosphere at the elevation of the top of the vortex and at the latitude of the principal east-west axis of the vortex. For a vortex that is in cyclo-geostrophic equilibrium, at all elevations z, the Coriolis and centrifugal accelerations due to the azimuthal velocity of the vortex are balanced by the horizontal pressure gradient within the vortex (Hassanzadeh *et al.* 2012). The top of the vortex, $z \equiv H$, is defined to be the location where the azimuthal velocity of the vortex is zero. Therefore at $z = H$, the horizontal pressure gradient within the vortex is zero. Therefore, in an ideal gas at the top of a vortex, $[T_{c'} - (\bar{T}|_{z=H})]/(\bar{T}|_{z=H}) = -[\rho_{c'} - (\bar{\rho}|_{z=H})]/(\bar{\rho}|_{z=H}) \simeq -H[(\partial\rho/\partial z)|_{c'} - (\partial\bar{\rho}/\partial z)]/(\bar{\rho}|_{z=H}) \simeq (H/g)(N_c^2 - \bar{N}^2)$, where the first approximate equality in this expression comes from using the first term in a Taylor series expansion to compute the density difference, and the second approximate equality comes from the definitions of the buoyancy frequencies. Thus, we can set the value of $(N_c^2 - \bar{N}^2)$ for the Jovian vortices equal to the observed value of $\frac{g[T_{c'} - (\bar{T}|_{z=H})]}{H(\bar{T}|_{z=H})}$.

Following Morales-Juberias *et al.* (2003), we set the upper elevation of the GRS to be at 140 mbar and use temperature measurements at that elevation. Satellite observations of the GRS (see figure 8 of Fletcher *et al.* 2010) give $[T_{c'} - (\bar{T}|_{z=H})]/(\bar{T}|_{z=H}) = -0.041 \pm 0.015$. Our estimate of the uncertainty of the Jovian temperatures of the GRS are based on the standard deviation of the data in the top-right panel of figure 8 in Fletcher *et al.* (2010). The values of $\sqrt{\bar{N}^2 - N_c{}^2}$ and N_c in table 1 for the GRS are based on these temperatures and uncertainties.

There have been no direct measurements of the temperature difference across the top of the Oval BA, but there were thermal measurements of the three White Oval vortices that existed at the same latitude as the Oval BA before it formed (and from which it formed). We use these White Ovals as a proxy for the Oval BA because it has been argued (de Pater *et al.* 2010) that the White Ovals were dynamically similar to the Oval BA and also of the same size and shape. For the Ovals, $[T_{c'} - (\bar{T}|_{z=H})]/(\bar{T}|_{z=H}) = -0.04 \pm 0.015$. For the Oval BA, we used the temperatures from figure 1 of Conrath *et al.* (1981). The uncertainty of those temperature measurements was not published, so our estimate of the uncertainty is based on the standard deviation of the values of $[T_{c'} - (\bar{T}|_{z=H})]$ of the three White Ovals at 140 mbar. The values of $\sqrt{\bar{N}^2 - N_c{}^2}$ and N_c in table 1 for the Oval BA are based on these temperatures and uncertainties.

To apply our equation (2.3) correctly to the Jovian vortices, which are not axisymmetric because they are embedded in strongly shearing east-west winds, it is necessary to consider the derivation of the equation for α based on cyclo-geostrophic balance shown in the companion paper (Hassanzadeh *et al.* 2012). The law for α can be obtained from the vertical hydrostatic equilibrium and cyclo-geostrophic balance applied to the east-west component of the force along the principal east-west axis of the vortex. The shearing east-west wind does not enter into this balance and therefore the Rossby number $Ro = V_\theta L/f$ that comes from this derivation and that appears in equation (2.3) for α (and that is reported in table 1), is based on the values of the north-south velocities along the east-west principal axis. Similarly the value of L that should be used in Ro (and that is reported in table 1) is that of the characteristic length scale of the pressure gradient along the east-west principal axis. Consistent with our approach of deriving the relationship for α by balancing the east-west component of the forces along the east-west principal axis, the measurements of $[T_{c'} - (\bar{T}|_{z=H})]$ that should be used in approximating $(N_c^2 - \bar{N}^2)$ of the Jovian vortices were made along the vortex's east-west principal axis.

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